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Catalog Maintenance of Low-Earth-Orbit Satellites: Principles of the Algorithm

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In Russia, the main source of information on orbital and ballistic characteristics of manufactured Earth satellites is the catalog of these objects maintained by the Space Surveillance System. Maintenance of this catalog is performed in real time by a complex automatic system, which includes a network of sensors and software tools for processing acquired data in automatic and interactive modes. The general structure and characteristics of this system for low-Earth-orbit satellites were first reported in February 1992 in Moscow at the conference on space debris and were published in 1993 [Khutorovsky, Z. N., "Satellite Catalog Maintenance," *Space Studies*, Vol. 31, No. 4, 1993, pp. 104–114 (translated from Russian)]. In continuation of that publication, the issues of the design of the system of software tools for automatic catalog maintenance are considered. Statistical decision theory is the basis for the synthesis of this system (complex algorithm). Analysis of the informational efficiency of this algorithm under real conditions is performed. It is demonstrated that the algorithm comprises two major components: primary determination of orbits and tracking. These are two continuous processes permanently interacting with each other. The character of this interaction is determined by the time–spatial pattern describing situations when the special "informativity" condition for the measurements is disturbed. A very general outline of these processes, following from the theory, is defined. Then the implemented algorithms for primary orbit determination and tracking are described. The methods of statistical decision theory under conditions of various types of a priori uncertainty are used in the synthesis of these procedures.

I. Introduction

We briefly describe the process of data acquisition and processing that is the basis of catalog maintenance for low-perigee satellites. Detection radars continuously scan certain domains of space and transmit the signals that are reflected from any resident object in space. When the reflected signal exceeds a certain threshold value, the mark or single measurement is generated. When a set of such excesses for different radar scans is generated for an object, the primary determination of its track normally occurs, i.e., track detection. When the determined track is further confirmed by new single measurements, radar tracking takes place. It is continued until the satellite either leaves the radar field of view or is lost. In the course of radar tracking, the single measurements are smoothed, and the measurement or observation of satellite orbital parameters is generated. This completes the control over the satellite orbit for one pass through one radar field of view. The duration of this process for low-perigee satellites ranges from several seconds to 10–20 min.

Immediately after acquisition, the measurements (orbital elements) are forwarded to the Space Surveillance Center (SSC) responsible for monitoring the satellites' orbital motion for the duration of their orbital life, which may be many years. For each measurement entering the SSC, the primary task is to determine

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if the satellite that produced it already exists in the catalog, i.e., the task of measurement assignment must be completed. As a result, the measurement becomes either correlated with a certain cataloged satellite or remains free (uncorrelated).

The correlated measurements update the orbital parameters of the satellites that have produced them, i.e., the task of orbit updating is accomplished. When the correct separation of measurements and the update of the orbital parameters for a certain satellite occur on a regular basis, the satellite is monitored or tracked. When the flux of the observations for a satellite ceases or becomes significantly sparse, the tracking of the satellite also may cease. Then break of tracking occurs.

The uncorrelated observations are involved in orbit detection (later we will designate this process as detection or primary orbit determination in contradistinction to radar tracking). The detection process is essential because the accuracy of the orbit determined on the basis of one pass through the radar's field of view (the accuracy of one observation) as a rule is not sufficient for reliable correlation of further measurements. The primary orbit determination integrates several measurements of the satellite track, and the resulting more accurate orbit will ensure efficient correlation of future measurements.

The new orbit (if reliable enough) may be either the orbit of a previously cataloged (tracked) but now lost satellite or the orbit of a new object. The task of the orbit identification algorithm is to determine which is the case. If the identification is successful, the data on the lost satellite are renewed, and its tracking is recovered. Otherwise, the primarily determined orbit is entered in the catalog as the orbit of a new satellite that will further participate in the tracking process. During the first phase of this process, the preliminary tracking, the origin of the satellite must be determined; i.e., the satellite identification task must be performed.

The described scheme of processing the measurements for maintenance of the satellite catalog is natural and seems to be quite reasonable. However, its efficiency is the issue to be investigated. Proper investigation must include the study of the initial data and the conditions for catalog maintenance. Then the most general formulation of this problem must be given, and the relevant solution suggested. Further, this solution is to be shaped into a realistic and efficient algorithm. All of these issues are the subject of this paper, which is a continuation of Ref. 1. Sections II and III consider the initial data and conditions. These are the characteristics of the sensors and of the features of the motion of the space objects monitored by these sensors. The structure of the algorithm for catalog maintenance is the subject of Sec. IV. Sections V–VII consider the components of this structure.

II. Observations and Their Errors

The main sources of the data for cataloging satellites are the detection radars that are located in the territory of the former Union of Soviet Socialist Republics in the near-boundary and central regions.[2] In a single observation, the radar receives target marks (single measurements) of range D, azimuth ε, elevation angle γ, and sometimes the radial velocity $\dot{D}$ in the local radar's coordinate frame. The origin of this coordinate frame is the conditional point of radar location. D is the distance between the origin of coordinates and the object; ε is the angle between the direction to the object and the direction $\boldsymbol{l}$ in the horizontal plane, which passes through the origin of coordinates; and γ is the angle between the horizontal plane and the plane determined by the position of the object and the direction $\boldsymbol{l}$. Vector $\boldsymbol{l}$ is normal to the main (central) direction of the radiation. It is directed to the right, as we look along the radiation. Note that the described coordinate frame is different from the usual topocentric spherical coordinate system. The angles are switched, and the reference direction for the readings of the azimuth is the main direction of radar radiation (not the direction to the north). This choice is convenient for the radars, which measure the azimuth with significantly better accuracy than the elevation angle. Actually, the angle ε is close to the azimuth directly measured by the radar, and the error of its determination is in fact the error in azimuth. On the other hand, the usual spherical azimuth depends on the elevation angle measured by the radar, and thus, the error of its determination is greater.

An observation is a six-dimensional vector of positions and velocities $\boldsymbol{x} = (D, \varepsilon, \gamma, \dot{D}, \dot{\varepsilon}, \dot{\gamma}) = (x_1, x_2, \ldots, x_6)$, resulting from the smoothing of single measurements within time intervals not exceeding 50–100 s. A radar observation x contains information on the orbit a of the satellite that has produced it because

$$\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{a}) + \delta\boldsymbol{x} \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{a})$ represents observation components as a function of orbital parameters and $\delta\boldsymbol{x}$ is observation error.

For development of the observations' processing algorithms, the accepted model of observation errors is of major importance; its accuracy determines the quality of the catalog maintenance. Observation errors not corresponding to the accepted model and exceeding tolerable limits will produce various undesirable effects. The first is that the observation may not be attributed to the proper satellite, thus becoming uncorrelated. In this case, we will have many debris observations in the file of measurements used for the primary determination of orbits, which results in a decrease of detection characteristics (increase of both detection interval and the probability of generating false orbits). The second is that the observation can disturb the orbit to which it is correlated. In this case, future observations of this object can be completely or partially missed, which results in a break of tracking.

The following model is used.

1) The quantity $\delta\boldsymbol{x}$ is

$$\delta\boldsymbol{x} = \delta\boldsymbol{x}_n + \delta\boldsymbol{x}_a \tag{2}$$

where $\delta\boldsymbol{x}_n$ and $\delta\boldsymbol{x}_a$ are normal and abnormal components, respectively, $\delta\boldsymbol{x}_n = (\delta D_n, \delta\varepsilon_n, \delta\gamma_n, \delta\dot{D}_n, \delta\dot{\varepsilon}_n, de\dot{\gamma}_n)$ and $\delta\boldsymbol{x}_a = (\delta D_a, \delta\varepsilon_a, \delta\gamma_a, \delta\dot{D}_a, \delta\dot{\varepsilon}_a, \delta\, dot\, \gamma_a)$.

2) Normal component $\delta\boldsymbol{x}_{i,n}$ is present in each radar parameter x_i. It has Gaussian distribution with the mean m_i and root-mean-square deviation σ_i $(i = 1, 2, \ldots, 6)$. Generally speaking, parameters m_i and σ_i are not known. However, they satisfy the conditions $m_i \leq m_{i,\max}$ and $\sigma_i \leq \sigma_{i,\max}$, where $m_{i,\max}$ and $\sigma_{i,\max}$ are known constants. The specific values depend on the type of the radar and normally lie within the intervals: $0.1 \div 1$ km for the range, $1 \div 100$ min for angular components, $0.001 \div 0.1$ km/s for radial velocities, and $0.01 \div 1$ min/s for angular velocities.

3) Abnormal component $\delta x_{i,a}$ satisfies the conditions $\delta x_{i,a} \gg \delta x_{i,n}$ and $|\delta x_{i,a}| \leq \delta x_{i,\max}$, where $\delta x_{i,\max}$ are known constants. The probability distribution is not known, as a rule. The error $\delta x_{i,a}$ is not always present in the measurement. It appears with certain probability $p_{i,a} \ll 1$.

Some characteristics of this model are noted as follows.

1) The systematic error in the value of m_i of the normal component can be comparable to the root-mean-square deviation σ_i and can even exceed it.

2) The probability of abnormal error in any of the radar components, as a rule, does not exceed 0.1.

3) Out of 64 possible combinations, 21 combinations of the components of the six-dimensional vector $\boldsymbol{x}$ can be abnormal simultaneously [to be exact, one or two components, with exception of the combination (D, ε), or three velocities].

4) The number of abnormal components is not more than three.

5) Parameters m_i and σ_i depend on time, coordinates in the radar coordinate frame, and the size of the observed object.

6) The values $m_{i,\max}$, $\sigma_{i,\max}$, and $\delta x_{i,\max}$ for each radar are chosen to ensure a 0.001 level for the error of the model. That means that not less than 99.9% of the real measurements have errors, corresponding to this model.

This model is used in making all the decisions in the process of catalog maintenance and is the key element of this procedure.

Various radar parameters are not equal in accuracy. The most accurate are the measurements of the range. Azimuth and elevation errors when transferred to linear values are usually much greater. The same relationship is valid for velocity components. Thus, for coordinate systems significantly different from the radar coordinate frame (for example, a rectangular frame or any sort of orbital elements), correlation between various parameters of observation occurs, and the model becomes more sophisticated.

The probability of the presence of abnormal errors does differ for various radar parameters. The most consistent is the range. Most susceptible to disturbances are the least accurate parameters: the elevation angle and the respective velocity. This also increases the sophistication of the error models for coordinate frames different from those in which the observations are obtained.

Thus, in the catalog maintenance procedures, the principal calculations involving observations are fulfilled in the radar coordinate frame. This decision makes the entire algorithm much more complex. Certainly the processing of observations after their transformation to orbital elements, for example, is easier. However, the arising informational losses are great.

Note that among the six orbital elements the following three can be accurately determined using one observation: orbital inclination i, longitude of the ascending node Ω, and the orbital period T. The root-mean-square deviation of the errors in determination of i, Ω, and T using one measurement is of the order of 1% of the measured value (for meter-sized satellites), i.e., approximately 1 deg for i, 1° for Ω, and 1 min for T. For decimeter-sized objects, the errors are greater, for example, the errors in T are on average five times greater.

III. Satellite Motion

The observed objects move in near-Earth space. For creation of a catalog maintenance algorithm, the model of this motion must be defined.

The orbital parameters $\boldsymbol{a}$ of a satellite satisfy a system of first-order differential equations of motion, $\boldsymbol{a}(t) = \boldsymbol{U}[\boldsymbol{a}(t-\tau), \tau]$. The function $\boldsymbol{U}(\boldsymbol{a}, \tau)$ is not known completely, and in practice approximate relationships $\boldsymbol{U}_0(\boldsymbol{a}, \tau)$ that define the specific prediction algorithm are used.

The prediction errors $\boldsymbol{V} = \boldsymbol{a}(t) - \boldsymbol{U}_0$ have two origins: incomplete accounting of perturbing factors and insufficient knowledge of the factors themselves. The errors arising due to the first reason can be virtually avoided, using accurate methods of approximate integration of the differential equations. Reduction of errors originating from the second reason is possible only by enhancement of our knowledge of the needed perturbating factors.

The major errors in the prediction procedures[3] are of the second type and are caused by insufficient knowledge of the atmospheric density. These errors are of random character, and to the first approximation can be described[4] by a Gaussian process with the mean equal to zero and the matrix correlation function $\boldsymbol{Q}(\tau_1, \tau_2) = M[\boldsymbol{V}(\boldsymbol{a}, \tau_1)\boldsymbol{V}'(\boldsymbol{a}, \tau_2)]$. The knowledge of statistical characteristics of the errors of atmospheric perturbations is not comprehensive. Their root-mean-square values depend on the altitude and the status of the atmosphere. In a calm condition for altitudes up to 500 km, their magnitude is of the order of 1–10% of the perturbation itself. Correlation interval ranges from several hours to several days.

Thus, the following model of objects' motion is used in the algorithms of catalog maintenance:

$$\boldsymbol{a}(t) = \boldsymbol{U}_0[\boldsymbol{a}(t-\tau), \tau] + \boldsymbol{V}[\boldsymbol{a}(t-\tau), \tau] \tag{3}$$

Correlation relationships between prediction errors for various moments for a certain satellite, as well as for various satellites, are of substantial importance. The technique used to take them into account will be considered in Sec. VI.

IV. General Composition of the Algorithm

In general, the task of catalog maintenance can be formulated as follows. Given the set of observations $\boldsymbol{X} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ produced by all sensors during the given time interval, determine the number of observed satellites k and estimate their parameters $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k$.

This is statistical decision problem. Reference 5 describes the mathematical technique used to find the solution. We will present only the final result without treating the mathematical details. The required solution obtains the minimum of the functional

$$\boldsymbol{\Psi}\big(\chi, \boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_{k(\chi)}\big) = \sum_{l=1}^{k(\chi)} \Psi_l(\boldsymbol{X}_l, \boldsymbol{a}_l) \tag{4}$$

where χ indicates a certain choice of assigning the observations to the satellites (arrangement), $\boldsymbol{X}_l = (\boldsymbol{x}_{l_1}, \boldsymbol{x}_{l_2}, \ldots, \boldsymbol{x}_{l_{m_l}})$ is the set of observations attributed to lth object according to arrangement χ, $k(\chi)$ is the number of objects in arrangement χ, $m_1 + m_2 + \cdots + m_k = n$ is the total amount of observations, and $\boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{a}_l)$ is a functional that describes the closeness between the chosen set of observations $\boldsymbol{X}_l$ and the parameters $\boldsymbol{a}_l$ of the lth object. The nonlinear maximum likelihood function $\boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{a}_l)$ is

$$\boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{a}_l) = [\boldsymbol{X}_l - \boldsymbol{F}_l(\boldsymbol{a}_l)]'\boldsymbol{M}_l^{-1}[\boldsymbol{X}_l - \boldsymbol{F}_l(\boldsymbol{a}_l)] \tag{5}$$

where $\boldsymbol{F}_l(\boldsymbol{a}_l)$ is the $6m_l$-dimensional vector, defining the relationship between the parameters of observations $\boldsymbol{X}_l$ attributed according to the arrangement χ to the lth object and the orbital parameters of the lth object for time t. Superscripts -1 and prime denote inversion and transposition of a matrix, respectively. Here

$$(\boldsymbol{M}_l)_{ij} = \boldsymbol{K}_{l_i}\delta_{ij} + \frac{\partial \boldsymbol{F}\big[\boldsymbol{a}_l\big(t_{l_i}\big)\big]}{\partial \boldsymbol{a}_l(t)}\boldsymbol{Q}_l\big(\tau_{l_i}, \tau_{l_j}\big)\frac{\partial \boldsymbol{F}\big[\boldsymbol{a}_l\big(t_{l_j}\big)\big]'}{\partial \boldsymbol{a}_l(t)} \tag{6}$$

where $\boldsymbol{M}_l$ is a $6m_l \times 6m_l$ square matrix, whose elements characterize the integral errors of the observations and the orbital parameters propagation for the times $t_{l_1}, t_{l_2}, \ldots, t_{l_{m_l}}$ of the observations $\boldsymbol{x}_{l_1}, \boldsymbol{x}_{l_2}, \ldots, \boldsymbol{x}_{l_{m_l}}$; $\boldsymbol{Q}_l(\tau_1, \tau_2)$ is the correlation function of the prediction errors along the track of the lth satellite; $\boldsymbol{K}_{l_i}$ is the correlation matrix of observation errors $\boldsymbol{X}_{l_i}$; and δ_{ij} is Kronecker's symbol. The functional (4) is minimized in all its arguments.

The trivial solution $k = n$, which provides an absolute minimum of $\boldsymbol{\Psi}$, equal to zero, always exists. Thus, without additional conditions the problem is mathematically degenerated.

The considered task has a specific feature that may be called the "informativity" condition for the observations. The condition is as follows. Using all of the information contained in the measurements of a certain satellite, its orbit can be determined with such accuracy that no alien observations will correlate with it. Formally this informativity condition can be presented as

$$\boldsymbol{\Psi}(\tilde{\boldsymbol{X}}, \boldsymbol{a}_{\min}) \ll \boldsymbol{\Psi}\big(\tilde{\boldsymbol{X}}, \boldsymbol{x}^*, \boldsymbol{a}_{\min}\big) \tag{7}$$

where $\boldsymbol{a}_{\min}$ is the point that minimizes the functional (5), which is determined using only the observations $\tilde{\boldsymbol{X}}$ corresponding to this object, and $\boldsymbol{\Psi}(\tilde{\boldsymbol{X}}, \boldsymbol{x}^*, \boldsymbol{a}_{\min})$ is the value of functional (5) for the point $\boldsymbol{a}_{\min}$ with an account of its own measurements $\tilde{\boldsymbol{X}}$ and alien measurement $\boldsymbol{x}^*$. If the informativity condition is satisfied, we need to minimize the functional (4) considering only the arrangements χ that correspond to this assumption. Thus, multiplication of orbits is avoided. The informativity condition ensures the existence of the unique (true) arrangement χ_{tr} that gives the conditional minimum $\boldsymbol{\Psi}$ dominating all of the others. Let the informativity condition be valid and the required solution be obtained for a certain moment. Then, when processing newly appeared observations, we have no need to change our decisions regarding the observations-to-satellites arrangement. Thus, the task is reduced to assignment of the new measurements to existing objects and subsequent updating of their orbits using the respective measurements.

For each new observation $\boldsymbol{x}$, the decision is made independently. Observation $\boldsymbol{x}$ is attributed to the object l^* that provides the minimum value $\min_{(\boldsymbol{a}_l)} \boldsymbol{\Psi}(\boldsymbol{X}_l, \boldsymbol{x}, \boldsymbol{a}_l)$, i.e.,

$$l^* = \arg\min_{l}\Big[\min_{\boldsymbol{a}_l} \boldsymbol{\Psi}(\boldsymbol{X}_l, \boldsymbol{x}, \boldsymbol{a}_l)\Big] \tag{8}$$

where $\boldsymbol{X}_l$ is the set of observations previously attributed to the lth object. If the number of satellites is k, then to solve the task we have to perform k minimizations of the maximum likelihood functional (5) and find the minimal one. This procedure is called the tracking algorithm. It is described in more detail in Secs. V and VI.

In practice the considered situation is typical, but not permanent. For some time intervals (t_{beg} and t_{end}) and domains D of the parameters, the informativity condition is not valid. This can be the result of the gaps between observations and the arrival of new satellites.

When local and short breaks of the informativity condition occur, the use of the tracking algorithm may result in false decisions. These decisions will consequently produce undesirable effects: missed detection of new satellites or breaks of tracking for cataloged objects. The following procedure is needed to avoid the mistakes. Separate the measurements $\boldsymbol{x} \in D$, $t \in (t_{\text{beg}}, t_{\text{end}})$ that do not satisfy the informativity condition. These measurements are accumulated without making any decisions until the time t_{end}, when the informativity condition will be satisfied again. Then the minimum of the functional (4) is determined using an exhaustive search through the set of measurements separated to satisfy Eq. (7). This procedure is called primary orbit determination. The observations used in this process are called uncorrelated. Techniques for selection of uncorrelated observations are described in Sec. VII.

The orbits determined on the basis of uncorrelated observations may be either orbits of untracked, uncataloged satellites or orbits of previously tracked cataloged satellites. This classification is done by a procedure called the orbital identification algorithm. The final stage, preliminary tracking of new satellites, results in final approval of their reliability, determination of their origin, and their transition to the status of regular tracking. The complete procedure of processing uncorrelated measurements and the newly determined orbits is called the detection algorithm.

Thus, the general algorithm of catalog maintenance consists of two main components: the tracking algorithm and the detection algorithm. These are two continuous processes, permanently interacting with each other. This interaction is determined by a spatial–temporal pattern, characterizing disturbances of the informativity condition for the observations.

V. Preliminary Correlation of Measurements and Objects

As mentioned in the preceding section, for each new measurement $\boldsymbol{x}$ we must first identify the satellite that produced it. The corresponding decision-making procedure was described in Sec. IV. The decision made by this procedure will be correct only when the informativity condition is satisfied. If this condition is not satisfied, the produced decision may be false, but can be changed in the course of further data processing. Therefore, this algorithm should be called preliminary correlation of observations with tracked satellites.

We have the following approximate relationship:

$$\min_{\boldsymbol{a}_l} \boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{x}, \boldsymbol{a}_l) \simeq \min_{\boldsymbol{a}_l} \boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{a}_l) + q_l(z) \tag{9}$$

where $z = \boldsymbol{x} - \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)$ is the difference between the observation and its estimation, based on the orbit of lth object; $\boldsymbol{f}_l(\boldsymbol{a}_l)$ is the relationship between the parameters of observation $\boldsymbol{x}$ and the orbital parameters $\boldsymbol{a}_l$ of the lth satellite; $\hat{\boldsymbol{a}}_l = \arg\min_{(a_l)} \boldsymbol{\Psi}_l(\boldsymbol{X}_l, \boldsymbol{a}_l)$ is the estimation of the lth object orbit, based on previous observations; and $q_l(z)$ is a quadratic form of the type

$$q_l(z) = z'\left[\boldsymbol{K} + \left(\frac{\partial \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)}{\partial \boldsymbol{a}_l}\right)\boldsymbol{R}_l\left(\frac{\partial \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)}{\partial \boldsymbol{a}_l}\right)'\right]^{-1} z \tag{10}$$

where $\boldsymbol{K}$ and $\boldsymbol{R}_l$ are covariance matrices of errors $\boldsymbol{x}$ and $\hat{\boldsymbol{a}}_l$ and $[\partial \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)/\partial \boldsymbol{a}_l]$ is the matrix of partial differentials of measurement parameters with respect to the orbital parameters for the point $\hat{\boldsymbol{a}}_l$.

Note that if the relationships describing the parameters of the observations as functions of orbital parameters and predicted orbital parameters are linear, and no time correlation of observation errors and prediction errors existed, we would have precise equality in Eq. (9). Because the actual situation is different no precise equality is in place. However, the smaller the observation and propagation errors, the more accurate is the equality in Eq. (9).

The matrix $\boldsymbol{K} + [\partial \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)/\partial \boldsymbol{a}_l]\, \boldsymbol{R}_l [\partial \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)/\partial \boldsymbol{a}_l]'$ is the covariance matrix of the vector z, thus the quadratic form (10) is the normalized distance between the observed and the evaluated (based on previously related lth satellite data) parameters of observation $\boldsymbol{x}$.

As follows from Eq. (9), the algorithm for making correlation decisions regarding observation $\boldsymbol{x}$, described in Sec. IV, is reduced to calculating quadratic forms (10) for all tracked cataloged objects and choosing the minimal of them. This procedure does not correspond to the real situation because it does not take into account breaks in the informativity condition (7) and the existence of abnormal errors.

To account for the first of the factors we provide for 1) the possibility of a negative correlation decision for the observation and the closest [in the sense defined by Eq. (10)] satellite and 2) the possibility of removing from the tracking process the satellites not updated for a long time.

We can realize provision 1 by making a negative correlation decision if $\min_{(l)} q_l[\boldsymbol{x} - \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)] > c(\alpha)$, where $c(\alpha)$ is the threshold, determined by the possible probability α of missing correlation ($\alpha \simeq 0.001$). These measurements become uncorrelated. Note that normally $c(\alpha)$ is chosen within the limits 10–100.

The accuracy of the position determination (defined by the errors of propagating the element set to the current moment) is analyzed periodically for all satellites. When these errors for an object exceed the given threshold, correlation of the observations with this object stops. (The threshold value is determined by the density of the satellites, observed in space. We choose it within the range 500–2500 km that corresponds to the 1–5 min range for errors in time.) These objects are then eliminated from the tracking process. Tracking of these objects is resumed only when they are identified as one of the newly detected satellites.

The decision function depends on parameter α, the probability of not correlating the measurement to its own satellite. Let us consider the choice of α. The result of missing the observation can be significant. The missed observation may never be correlated to its satellite. If it becomes uncorrelated it will become a wayward measurement, producing difficulties in the detection of new objects. If many measurements of this type occur for a certain satellite the break of its tracking can occur. Therefore, the smallest value of α must be the goal. In fact, the value of α has the lower limit, determined by the accuracy of the model of observation errors. The observation error model, described in Sec. II has the error ≈ 0.001. Hence, we will require $\alpha = 0.001$.

In Sec. II, we mentioned that $\approx 10\%$ of the measurements are abnormal. Using $\alpha = 0.001$ only 1% of them can be missed in the measurements-to-satellites correlation process. That means that the gates used in the decision function must be broad enough. However, the broader the gates, the greater is the probability β of assigning the measurement to an alien (wrong) object. The false correlation can happen when the informativity condition is disturbed, for example, by the presence of observations generated by the satellites that are not detected yet, but are close to the tracked objects. However, the losses due to incorrectly correlated measurements are as a rule smaller than the losses caused by the missed observations because the individual incorrectly correlated measurements can be selected further in the course of tracking (see Sec. VI.D).

Now we will consider the issues related to the account of the observation errors model, described in Sec. II. The minimax approach is the basis here.[6] The rationale for the use of this approach is as follows. The statistical description of abnormal errors is not known; we are aware only of their maximal values. However, even under these conditions the correlation algorithm must ensure that the probability of missing α is less than required. Thus, the requirement of small α is to be fulfilled for any abnormal errors, including their maximum values that produce the greatest losses in the quality of the decision making process. These greatest losses are minimized by optimizing the decision function. This is the task for the minimax approach.

The foundations of the statistical decision theory foundations[6] are not described here; the decision function of the algorithm will be described. Let $\boldsymbol{N} = \{\boldsymbol{x}_n\}$ and $\boldsymbol{A} = \{\boldsymbol{x}_a\}$ be the sets of possible combinations of normal and abnormal components $\boldsymbol{x}_n$ and $\boldsymbol{x}_a$ (elements of these sets), the specific combinations of normal and abnormal components of the observation vector $\boldsymbol{x}$. As we mentioned in Sec. II, the number of elements in the sets $\boldsymbol{N}$ and $\boldsymbol{A}$ is equal to 21, and the number of components in each element $\boldsymbol{A}$ does not exceed 3. Let $\boldsymbol{L} = \{l\}$ be the set of objects for which the differences z_i from the parameters x_i of the observation x are tolerable, i.e.,

$$|z_i| \le k \cdot \sqrt{\sigma_{x_i}^2 + \sigma_{x_{i|l}}^2} + \delta x_{i,\max}, \qquad i = 1, 2, \ldots, 6 \tag{11}$$

where σ_{x_i} and $\sigma_{x_{i|l}}$ are the root-mean-square deviations of normal errors in the measured and determined according to the lth object orbit parameters of the observation $\boldsymbol{x}$, $\delta_{x_i,\max}$ are the maximal values of possible abnormal components of the errors in the observation $\boldsymbol{x}$, and k is the constant, defined by tolerable probability of missing the correlation. (The value $k = 3$ ensures the required probability of missing.)

Then the observation $\boldsymbol{x}$ is considered uncorrelated if

$$\min_{l \in \boldsymbol{L}} \min_{\boldsymbol{x}_n \in \boldsymbol{N}} q_l(\boldsymbol{x}_n - \boldsymbol{x}_{n|l}) > c(\alpha, \boldsymbol{n}^*) \tag{12}$$

where $\boldsymbol{x}_{n|l}$ is the estimation of the normal components $\boldsymbol{x}_n$ based on the parameters of the lth object; $c(\alpha, \boldsymbol{n}^*)$ is the threshold depending on required probability of missing the correlation α and the dimensionality of the normal components vector $\boldsymbol{n}^*$ and corresponding to the first minimum of Eq. (12) [$c(\alpha, \boldsymbol{n}^*) \approx 10\boldsymbol{n}^*$]. Otherwise, the observation $\boldsymbol{x}$ is correlated to the object l^*

$$l^* = \arg\min_{l \in \boldsymbol{L}} \max_{\boldsymbol{x}_n \in \boldsymbol{N}} q_l(\boldsymbol{x}_n - \boldsymbol{x}_{n\,|\,l}) \tag{13}$$

To determine this decision function for one object, 21 matrices with dimensions from 3×3 to 6×6 (according to the number of possible combinations of normal components) must be inverted; this is not convenient. However, the procedure can be essentially simplified without noticeable losses in efficiency. The greatest difficulty to overcome is the essential correlation between the components of vector $\boldsymbol{z}$ caused by significant errors in time due to the propagation of the orbit to the moment of observation. For such cases these correlations must be taken into account, otherwise the errors of false correlation β will significantly increase.

The accounting for the correlation is done as follows. When we considering the space of the correlation parameters $\boldsymbol{z} = \boldsymbol{x} - \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l)$, the nonlinear transformation $\boldsymbol{z} \Rightarrow \tilde{\boldsymbol{z}}$ of stochastic vector $\boldsymbol{z}$ is fulfilled when the components of the resulting vector $\tilde{\boldsymbol{z}}$ become slightly correlated. This transformation is the removal of the error in time $\delta\tau$ from the parameters of the objects, using the most informative component of the observation (as a rule, range).

The transformation is fulfilled as follows. The parameters of the satellite aspiring to the observation $\boldsymbol{x}$ are propagated to the point where the calculated and the measured values of the most informative component, i.e., the component providing the most accurate estimation of $\delta\tau$, of observation $\boldsymbol{x}$ coincide. In case we have several such points within the revolution, we will choose the one that will provide the minimum residual for the other informative component of the observation (usually, if the most informative component is the range, the second one is the azimuth).

Denote $\delta\hat{\tau}$ the difference between the time to which we are propagating and the time t of the observation $\boldsymbol{x}$ (estimation of the time error in orbital parameters for the moment of the observation), and denote $\boldsymbol{f}_l(\hat{\boldsymbol{a}}_l, \delta\hat{\tau})$ the components of the observation, calculated using orbital parameters for the time $t + \delta\hat{\tau}$. Then the vector $\tilde{\boldsymbol{z}}$ is

$$\tilde{\boldsymbol{z}} = (\boldsymbol{x} - \boldsymbol{f}_l(\hat{\boldsymbol{a}}_l, \delta\hat{\tau}), \delta\hat{\tau}) = (\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_6) \tag{14}$$

The vector $\tilde{z}$ is six dimensional because one of its components, used for removal of the temporal error, is equal to zero and is not considered anymore. The decision function [Eqs. (12) and (13)] is now essentially simplified.

The observation $\boldsymbol{x}$ and the lth satellite are considered correlated when any of the combinations of inequalities

$$\tilde{z}_i^2 \le k^2\left(\sigma_{\tilde{z}_i}^2 + \sigma_{\tilde{z}_{i\,|\,l}}^2\right) \tag{15}$$

corresponding to the possible set $\boldsymbol{x}_{\text{nor}}$ of normal components are satisfied, and the inequalities

$$|\tilde{z}_i| \le \delta\tilde{z}_{i,\max} + k \cdot \sqrt{\sigma_{\tilde{z}_i}^2 + \sigma_{\tilde{z}_{i|l}}^2} \tag{16}$$

are valid for other abnormal components $\boldsymbol{x}_a$ [with respect to Eqs. (15) and (16) for all of the parameters, except $\delta\hat{\tau} = 3$ and $\delta\tilde{z}_{i,\max} = \delta\tilde{x}_{i,\max}$; for $\delta\hat{\tau}$ in Eq. (15), k ranges from 3 to 100 depending on the assessment of the status of the atmosphere (see Sec. VI.F), and the right side of Eq. (16) is 300 s]. If an observation is correlated to several satellites, we choose the satellite corresponding to the minimum of the quadratic form

$$\tilde{q}_l = \sum_{i=1}^{6} \frac{\tilde{z}_i^2}{\left(\sigma_{\tilde{z}_i}^2 + \sigma_{\tilde{z}_{i\,|\,l}}^2\right)} \tag{17}$$

calculated using all six components of the vector $\tilde{z}$.

The presented algorithm is simple and physically clear. Finally we will treat some issues of its implementation in practice. Direct calculations according to this procedure require a lot of computation. To reduce operations, the rough selection of satellites surely incapable of producing the measurement $\boldsymbol{x}$ is done without requiring precise propagation. First, we use the inclination i and the longitude of the ascending node Ω calculated using the measurement (first step of the correlation). Then, we fulfil the interval gating for the most accurate parameters of the observation using a rough prediction of the candidate orbit to the moment of the observation (second step of the correlation). The propagation procedure used here is the polynomial propagation with a transformation (under Kepler's approximation) into radar coordinates D, ε, and γ. Note that the coefficients of this polynomial for each satellite are calculated after the update of its orbit and are stored in the catalog. The technique of this calculation is as follows. The evolution of the mean elements λ, L, θ, Ω, e, and ω is approximated by the polynomials of given order within the interval Δ. The value $\Delta \approx 1{-}10$ days and depends on the decline of orbital period per revolution. The order of the polynomials is: the fourth in λ, the third in L and e, and the second in θ, Ω, and ω. The approximation uses the values of the mean orbital elements (calculated using precise prediction) for several moments within the interval $(t, t + \Delta)$, where t is the epoch of updated elsets. The approximation is fulfilled using the least-squares technique. Last, to the third step of the correlation with the observation $\boldsymbol{x}$, only the satellites selected at the two preceding steps are forwarded, where the earlier described minimax decision function and precise prediction is used.

Under the condition that the required probability of missing the correlation is 1–2 orders of magnitude less than the frequency of the rough measurements, this scheme is not fast enough. The following is used to make it more effective. Correlation is fulfilled in three passes. For the first pass we use narrow gates in i and Ω, of the order of $1 \div 2$, not suitable for correlating rough measurements. But $\approx$90% of the measurements will be correlated. Uncorrelated measurements ($\approx$10% of the total amount) are forwarded to the second pass, where they are selected in i and Ω using gates of the order of $10 \div 20$, sufficient for the majority of rough measurements. Measurements uncorrelated at the second pass ($\approx$1% of the total amount) are subjected to the third pass, with no selection in i and Ω.

This is the general composition of the algorithm of preliminary correlation of observations with the tracked cataloged satellites. Recall that if the informativity condition is not valid, the obtained decision may not be true. The mistake can be corrected later when new observations of the satellite miscorrelated with this observation arrive. The relevant procedure is described in the next section. However, in case the observation produced by the cataloged object failed to correlate any of them, it can find its parent satellite only via the detection process described in Sec. VII.

VI. Updating of Orbits

Correlated observations are used to update the current element sets of the satellites they are attributed to. This process is now discussed.

A. Minimized Function

The maximum likelihood functional (5) used for estimation of the elsets on the basis of measurements is a quadratic form based on the differences between the observed and calculated parameters. The matrix of this quadratic form is not diagonal due to the time correlation of the perturbations, which is not accounted for in the propagation of orbits.

Direct account of the correlations according to Eq. (5) is bulky and inefficient; however, disregarding them will produce information losses. Theoretical and experimental studies resulted in the following reasonable version: the nondiagonal components of matrix $\boldsymbol{M}$ in Eq. (5) are set to zero, and the effect of the correlation is provided by processing not only the observations, but the previously obtained element sets as well. The resulting minimized functional is

$$\Phi(\boldsymbol{a}) = \sum_{p=1}^{m}[\boldsymbol{x}_p - \boldsymbol{f}_p(\boldsymbol{a})]'(\boldsymbol{K}_p + \tilde{\boldsymbol{K}}_p)^{-1}[\boldsymbol{x}_p - \boldsymbol{f}_p(\boldsymbol{a})]$$

$$+\sum_{q=1}^{r}[\boldsymbol{a}_q - \boldsymbol{a}_q(\boldsymbol{a})]'\boldsymbol{P}_{\boldsymbol{a}_q}[\boldsymbol{a}_q - \boldsymbol{a}_q(\boldsymbol{a})] \tag{18}$$

where $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m$ are the observations for the moments $t_1 \leq t_2 \leq \cdots \leq t_m$ correlated to the object; $\boldsymbol{f}_p(\boldsymbol{a})$ is the relationship between the parameters of observation $\boldsymbol{x}_p$ and the object's parameters $\boldsymbol{a}$; $(\boldsymbol{K_p})_{ij} = \sigma^2_{x_{p,i}}\delta_{ij}$, where $\sigma^2_{x_{p,i}}$ is the variance of errors for the ith component $\boldsymbol{x}_p$; $(\tilde{\boldsymbol{K}})_{ij} = \tilde{\sigma}^2_{x_{p,i}}\delta_{ij}$, where $\tilde{\sigma}^2_{x_{p,i}}$ is the variance of the orbit's prediction error for the time t_p for the ith component parameter $\boldsymbol{x}_p$; and $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_r$ and $\boldsymbol{P}_{\boldsymbol{a}1}, \boldsymbol{P}_{\boldsymbol{a}2}, \ldots, \boldsymbol{P}_{\boldsymbol{a}r}$ are a priori (previously evaluated or known from other sources) estimates of the elsets for the times $\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_r$ and their weighting matrices. The number r of these estimates usually is 1. ($r = 0$ is not used and $r > 1$ occurs when the analyst introduces additional data on the object).

The vector $\boldsymbol{a}$ of orbital parameters is a seven-dimensional vector $\boldsymbol{a} = (\lambda, L, \theta, \Omega, h, k, s)$, where $(\lambda, L, \theta, \Omega, h, k)$ is the element set (see Sec. VI.B) and s is the ballistic coefficient coordinated with the used model of the atmosphere. The vector $\boldsymbol{a}$ is timed to the moment t_m of the last observation.

The functional (18) is a sum of the squares of the normalized differences between observed and a priori data and, thus, is the function of the least-squares method. Distinct from the typical least-squares method is that in the calculation of the weights (coefficients of the squares of the differences) the errors of the orbit prediction are included along with the observation errors. These errors are accounted for by correcting the variance of the measured parameter by the value of the variance of the neglected prediction procedures perturbations.

Prediction errors increase as the propagation interval increases. This results in the decrease of the observations' weights in Eq. (18) when they grow older. The result is an estimation algorithm with effective finite memory. Thus, there is no need to use outdated measurement data for orbits' updating.

The previous estimate of the orbital parameters is usually used as the a priori data. The weighting matrix of this estimate is diagonal, with the elements being the inverse of the variances. Certain constants provide lower limits for the elements, whose values depend on the decline of the orbital period per revolution and some other factors, determined in the course of the tracking process (see Secs. VI.E and VI.F).

B. Prediction of Motion

Prediction of the motion, or calculations of the orbital parameters for arbitrary times, based on their values known at epoch is the major calculation procedure, not only in the updating algorithm, but in the catalog maintenance procedure as a whole. The accuracy and speed of the propagators determine the overall feasibility and characteristics of the algorithm.

The prediction errors of the orbit prediction method must not exceed the maximum of two values, the observation errors and the potential real prediction errors, caused by the uncertainty in the major perturbations. Also, the computation time must be relevant to the needs.

It is difficult to create the universal algorithm satisfying these requirements for all possible situations. Therefore a package of prediction procedures was developed. The algorithms used for low-Earth-orbit satellites and their characteristics are treated in detail in a previous paper.[3] Here we present a brief description.

For satellites with small atmospheric drag, the propagation is fulfilled by the analytical algorithm. The algorithm uses reduced Delaunay's elements $\lambda, L, \theta, \Omega, h$, and k, obtained from the standard Kepler's elements M, a, i, Ω, e, and ω according to: $\lambda = M + \omega$, $L = \surd(\mu a)$, $\theta = \cos i$, $\Omega = \Omega$, $h = e \sin \omega$, and $k = e \cos \omega$, where μ is the gravitational constant. The propagator includes all significant zonal and tesseral geopotential harmonics up to eighth order (8×8 field) and the static model of the atmosphere with parameters, depending on the levels of solar activity according to the Russian model of the atmosphere.[7] The formulas are derived using Brouwer's method. Thus, periodic perturbations from the zonal harmonics are defined with the error $\approx c^3_{20}$, secular with the error $\approx c^3_{20}t_{\mathrm{pr}}$ and tesseral perturbations with the error $\approx e^2$, where c_{20} is the second zonal harmonic, t_{pr} is the propagation interval, and e is the orbit eccentricity. Time is used as the independent variable. The algorithm is presented in compact recurrent form, having no singularities for small eccentricities. In the vicinity of the critical inclination ($i \approx 62.3$ deg), periodic perturbations are approximated by secular. An ongoing calculations mode is implemented, saving computation time by using once calculated and remembered secular and long-periodic coefficients for propagating the same elements for various moments. This is the primary mode used for updating the orbits.

Propagation of near-circular satellites ($e < 0.1$) with significant atmospheric perturbations is performed by the numerical–analytical algorithm using Delaunay elements and an 8×8 geopotential, and atmospheric drag is defined by a dynamic model of the upper atmosphere.[7] The algorithm is used for the entire time the satellite is in the Earth's atmosphere, except for the last days of orbital life. The algorithm is based on the third-order Runge–Kutta integration of the Krylov–Bogolyubov averaged motion equations for the doubly Brouwer-averaged orbital elements. Time is used as the independent variable. The integration step varies from 1 to 5 days depending on the time remaining in orbit. Within the integration step, the elements are calculated using polynomial interpolation. To reduce the computation time for calculating the atmospheric density, rather simple but sufficiently accurate approximations are used together with special ongoing calculation modes.

Propagation of the motion for orbits with $e < 0.1$ for the last days in orbit up to the moment of re-entering is performed using the numerical algorithm with a 6×2 geopotential and atmospheric drag according to the Russian model.[7] Adams method for Delaunay's osculating elements is used. The major parameters of the procedure (the integration step, the composition, and the order of accuracy) are chosen empirically.

C. Minimization Technique

The search for the minimum of the nonlinear functional (18) is an important calculation process. It is used not only in orbit updating, but in other parts of the general algorithm, particularly in the detection procedures.

A specially developed combination of two classical techniques, the Gauss–Newton method and the steepest descent method, is used.[8] The algorithm has three levels (zero, the first, and the second), each of them using a specific technique for defining the step. The transitions between the levels are performed using the scheme $0 \to 1 \to 2 \to 0 \to \cdots$. Transition to the next level occurs when the decrease of the functional at the current level is less than 10. Transition to the zero level always occurs after one step at the second level (disregarding the initial and the final values of the functional). Iterations always start from the zero level. Here either the Gauss–Newton method is chosen or the fastest descent according to the step of Gauss–Newton. The descent is prescribed when the normalized step exceeds the threshold value. (Each of the parameters is normalized to the value equal to the expected maximum error of its determination in the initial approximation.) Otherwise, the step of Gauss–Newton is performed. When the descent method is chosen, three steps are always fulfilled. For the first level, the step of Gauss–Newton is always used. At the second level, the Gauss–Newton step is made, followed by three descents (corrective descents).

The investigations revealed that in the scope of our task the corrections of the Gauss–Newton step by several sequential descents often assist the convergence when Gauss–Newton and the fastest descent techniques fail to be efficient. The mathematics of this inefficiency can be described as follows. Within the area of our interest, the functional has one local minimum. The matrix of the second partial derivatives of the minimized functional is poorly defined, and the directions of Gauss–Newton and the descent lie within the subspace of the eigenvectors of this matrix, corresponding to the least eigenvalues. Actually, when using the corrective descents, the following situation occurs. The level curves of the functional have a ravine structure, and the long step of Gauss–Newton takes us out of the unfortunate domain. This step transfers us to the other side of the ravine from which we can descend efficiently.

At all of the levels, the size of any step in the chosen direction (descent or Gauss–Newton) is refined by minimization along this direction. Such one-dimensional minimization is performed in a set of iterations. The procedure is as follows:

The evolution of the functional along the chosen direction is approximated by a third-order polynomial. The polynomial is determined using four points: the values of the functional and its derivative along the chosen direction for the initial and the final points of the calculated step (of descent or Gauss–Newton). The point of the minimum of the polynomial is determined within the step. The algorithm is rather simple and in fact means the solving of the quadratic equation.

Experiments have shown that often the abrupt variation of the functional along the chosen direction is poorly approximated by the third-order polynomial. Therefore, the minimum point is determined iteratively using the polynomial constructed for the smaller interval. For the next iteration, the initial point of the interval remains the same, and the minimum point of the polynomial is the end of the interval. A maximum of three such iterations are made. The iterations also terminate if the minimum is attained for one of the ends of the interval.

The iterations stop in any of the following five cases:

1) The normalized Newton-Gauss step becomes smaller than the threshold and the relative variation of the functional does not exceed the given value (we arrived at the minimum).

2) The certain number of iterations is completed and the absolute value of the functional is less than the threshold value [we obtained the orbit that inscribes (corresponds to) all of the measurements participating in the updating process].

3) The maximum possible number of iterations is completed but the conditions 1 or 2 are not satisfied (the divergence).

4) For the fourth time we come to the second level (we are stuck).

5) In the iterative process, the values of the parameters have significantly deviated from the a priori values, and the calculated values for certain variables are out of possible range (we left the right way).

Only for the first two cases is the solution considered obtained.

The initial orbital elements used in the iterative process of the minimization of the functional (18) are the parameters defined by the last updating. For the great majority of cases, their accuracy is sufficient to provide convergence of the iterative process. However, sometimes convergence is not achieved. For these cases, the initial parameters are those obtained by minimization of Eq. (18) in three (out of seven) components of the vector $\boldsymbol{a}$: the orbital elements λ and L and the matching ballistic coefficient s. Note that this is not sufficient for orbit detection where a special algorithm must be used (see Sec. VII.D).

For the majority of cases, calculation of the partial derivatives of the observations' parameters with respect to the orbital parameters needed for determination of the step[8] is performed only for the first iteration using the analytical formulas including the effect of the second zonal harmonic and static atmosphere model. In the course of further iterations, they are not redefined. When the number of observations m participating in the updating and the respective time interval $t_m - t_1$ are less than the prescribed values, the partial derivatives are redefined for each step. For the last day of the orbital life of a satellite (before reentry), they are numerically calculated for each iteration.

D. Selection of Abnormal and Alien Observations

Breaks in the informativity condition and the existence of abnormal observation errors complicate the orbit updating. Disregarding these factors will inevitably result in disturbances in the tracking process.

Abnormal and alien observations are identified using the multipass minimization of the function (18) and selecting in each pass the abnormal components of all of the observations using normalized differences between the observed and the estimated values. We will treat this algorithm in more detail because it is one of the key aspects of the tracking procedure.

Substantially abnormal components of the observation can disrupt the orbit in the minimization process, thus complicating the selection. Therefore they are identified prior to minimization by comparing the squares of the normalized differences z_i/σ_{zi} obtained during the preliminary correlation of measurements and objects to the high threshold (of the order of 100–10,000). At the first pass, all of the previous, i.e., those which participated earlier in updating of the orbit, observations are used with the weights (coefficients of differences' squares) obtained by the preceding update of the orbit; i.e., the weights of those components of the observations that were considered to be abnormal at the last updating are set to zero. It is better to reduce the weights of the abnormal components according to the a priori data on the maximum values of the abnormal errors. However, the roughest parameters of the observations are most frequently disturbed. Thus, the chosen technique does not produce informational losses. In addition, the weights of those components of new, i.e., not participated in the updating of orbit, observations that did not pass the earlier mentioned minimization rough selection are set to zero as well. The other components of the new observations participate in the first pass of minimization with their weights calculated according to Eq. (18), under the assumption that they are not abnormal.

Before each next pass the possible abnormality of all of the components of all of the measurements (the old as well as the new ones) is tested by comparing the square of the normalized difference [corresponding additive in Eq. (18)] to the point of the minimum to the low threshold (equal to 10). The weights of thusly detected abnormal components are set to zero, and previously mistakenly excluded normal components are reincorporated into the process with restored weights. If the current minimization reveals that all of the weights are chosen correctly the passes end.

The described algorithm is time consuming. Decisive reasons to choose it were provided by the results of the modeling, which revealed that this technique is more efficient than other robust procedures. It should be noted that if the percent of abnormal and alien observations is large ($\approx$20–40% and more) and the initial orbit is rough, the efficiency of this procedure declines abruptly and other robust methods can be more effective. However, these situations are very rare in the practice of satellite tracking.[1]

Obviously, more efficient techniques can exist only when the probability of the true selection of abnormal components is less than 1. Hence, they exist only when the share of abnormal components exceeds a certain threshold (usually 0.2–0.4). The limitation of the used technique for this case is that the weights of the undetected abnormal components are overestimated and the weights of some of the normal components may be set to zero. More efficient estimates for this case can be obtained, for example, using the following technique: the weights of the observations for each iteration are not constant, but are certain functions of the residuals, which rapidly decrease with their increase.

Further decision making depends on the reliability of the obtained result. The reliability criteria must provide not only effective selection of abnormal and alien measurements, but the future steady tracking of the satellite (if the informativity condition is satisfied) as well. These criteria are universal, meaning that they are used not only in tracking, but in making the principal decision regarding the existence of the new orbit as well. Thus, the reliability criteria are the key elements of the whole catalog maintenance algorithm.

Without detailed treatment of the foundations (certain considerations are presented in Sec. VII.A), we will formulate the basic rule.

The solution is reliable when 1) it is obtained (see the preceding section) and 2) the set of observations inscribed into the orbit is complete.

The observation is inscribed into the orbit in the case where three of its most accurate components have nonzero weights. The set of observations is complete when it contains at least three observations for three different revolutions. The features of complete sets of observations are treated in more detail in Sec. VII.A. Updating occurs when the solution is reliable and at least one new measurement is inscribed into the orbit. If the updating took place, the selection of alien observations is fulfilled. To do it, all of the observations are correlated to the obtained orbit. The decision function is the same as the procedure used for preliminary correlation of the measurements and objects (see Sec. V). Selection of the alien observations is needed because, if the informativity condition is disturbed, mistakes can occur in the preliminary assignment of the observations to tracked satellites. This selection is possible because preliminary assignment of the measurements uses extrapolation of the orbits for calculation of the residuals. After the orbit updating, the interpolation is used. Interpolation errors are smaller, and, thus, the gates used for making decisions according to Eqs. (15) and (16) after updating are also smaller.

To avoid mistakes in the decision making caused by unpredictable changes of the satellites' parameters (see Sec. VI.E), a decision regarding correlation of certain measurements is made only when a more recent observation, inscribed to the orbit, exists.

If the observation is not considered alien, but some of the weights of the components are nullified, it is considered related to the object and its zero-weight components abnormal. Abnormal observations take part in subsequent updating of the orbit, and previous decisions may be changed in the course of this process (new anomalies may be found, the old ones may vanish, the observation may become alien).

Alien observations are subjected to one more preliminary correlation with all of the tracked cataloged objects, excluding the considered one and those with regard to which similar decisions were made. Such an observation may perform a rather sophisticated journey before it is finally attributed to some satellite or considered uncorrelated. Usually such situations occur at the initial stage of orbit detection for the fragments of a multielement launch or a breakup or in the case of small-sized objects' conglomeration.

As we have already mentioned, the reason for the appearance of alien observations is the disturbance of the informativity condition. In the situations treated earlier, the tracking procedure is capable of overcoming the consequences of such disturbances and the breaks of tracking are avoided. In practice this situation is typical. More rare are the situations when no reliable solution is obtained in the updating of an orbit. Then break of tracking occurs. This satellite is then excluded from the tracking process (correlation with it is prohibited), and its observations are forwarded to recorrelation with other objects. Some of these observations may be further correlated with other satellites, and the others are considered to be uncorrelated and transferred to the detection procedure.

If the observations transferred to the detection process will be incorporated to certain preliminary determined orbit, the orbit of the satellite excluded from the tracking process will be renovated (identification by number), and prohibition of correlation will be cancelled.

The failure in making reliable decisions is usually caused by disturbances of the informativity condition. The tracking algorithm can not overcome these disturbances and is not able to select alien measurements from the own ones. Break of tracking occurs, and the object is transitioned to the detection process, which is a more powerful procedure, providing the quest for measurements produced by one and the same object by means of exhaustive search.

E. Unpredictable Variations of Parameters

During satellites' tracking their parameters may be subjected to variations not adequately accounted for by the used prediction algorithms. These variations may be caused by the following: 1) satellite's engine thrusting, 2) changes of orientation, 3) satellite fragmentation, 4) powerful geomagnetic disturbances, 5) decay of the satellite during re-entering, and 6) influence of disregarded perturbations (light pressure and lunar and solar gravitational perturbations).

To provide tracking under these conditions means that the following tasks must be solved: 1) assurance of the observations' correlation stability, 2) timely detection of changes in the parameters and determination of their origin, and 3) assessment of the characteristics of the variations and the a posteriori orbital parameters.

Let us describe the relevant procedures.

Correlation of the observations is when parameter changes result from respective widening of the gates for time difference δ_τ. For the maneuvering objects, the correlation decision function is modified as well. In addition to the measurements satisfying the criteria of Sec. V, with such satellites we correlate the observations, uncorrelated (with regard to all tracked objects) but differing in their parameters from the object's parameters by not more than a specified amount, defined by maneuvering capabiloities and the time passed since the maneuer was performed.

Detection of the change of the parameters is performed after selection of the abnormal and alien observations, on the basis of the number of last observations in a row not inscribed into the orbit and the time interval of these observations. The parameters of the decision function are chosen experimentally according to the requirements for the frequency of false detections and missing the changes and the need to provide steady tracking of the satellite after its parameters have changed. These requirements are different for different satellites and for various factors responsible for variation of the parameters.

The cause of the occurred change of the parameters is determined after the detection of this fact. The foundations of the algorithm are as follows. Only some satelites are capable of maneuvering. The objects, subjected to significant propagaton errors due to disregard of lunar, solar, and light pressure perturbations can be distinguished by orbital parameters and their evolution. The changes of orietation are possible either for known operational satellites or for the objects close to reentry. However, for other satellites subjected to atmospheric drag, we do not reveal simultaneous variations of parameters. Finally, under geomagnetic storm conditions, simultaneous one-way varation of ballistic coefficient estimations occurs for all objects influenced by atmospheric drag. In particular, it provides for timely detection of a geomagnetic storm, the assessment of unpredictable temporal variations of the atmospheric density, and for the possibility of accounting for this variation in the calculations of the gates for preliminary observations-to-satellites assignment. Other reasons for the changes of parameters can be identified by special analyst efforts.

If the change of parameters is detected and its possible origin is identified, the evaluation of its characteristics and new orbital parameters is fulfilled. To do this, based on postvariation observations (the set of last not-inscribed observations in a row up to the first inscribed), the prevariation orbit, and the a priori data on the magnitude of variation, the new orbital parameters are estimated using minimization of function (18). A posteriori parameters of the alteration, in particular, maneuver time and momentum, are evaluated comparing the resulting orbit to the last one obtained before the change.

Some specific features of the algorithm are as follows. The most important are the issues of using the a priori data and the problem of obtaining a reliable solution. Usually the estimation, based on observations prior to the parameter's alteration, is used as the vector of a priori parameters in Eq. (18). The weights of the a priori data for maneuvering satellites depends on available data characterizing a possible maneuver and for not maneuvering, on the origin of alteration, and on its characteristics, in particular, the magnitude of unpredictable variation of the atmospheric density.

To obtain the reliable solution after the change of parameters (especially following the maneuver) is rather difficult due to substantial errors in initial values (a priori parameters), few observations, and increased probability of the presence of alien measurements. Therefore, to select alien observations prior to minimization, time differences with the a priori orbit and the data on the possibilities of the observations are used.

After minimization of Eq. (18) using the selected observations, the reliability condition is tested. In general the reliability criteria are similar to those described in Sec. VI.D, taking into account that the a priori orbit is considered to be an additional measurement.

When the reliable solution after change of parameters is obtained, further updating of orbit is fulfilled in the conventional manner. If the reliable solution after alteration can not be obtained, the tracking of the object is recovered via the detection process according to the general procedure described in Sec. VI.D. Only postalteration observations are returned to the preliminary correlation.

Finally, note that the not intensive variations of parameters may not be detected. In these cases, conventional tracking is carried on automatically taking into account specific features described in the next section.

F. Adaptation for the Errors of Observations and Prediction

A statistical description of the errors in the measurements and prediction is not completely known. Sometimes the uncertainty is rather great. For example, reliable knowledge exists only for the maximum values of abnormal errors. In other cases, the level of uncertainty is lower. The parametric character of uncertainty is possible, i.e., the type of the distribution may be known, but it comprises unknown parameters that can be considered the interfering parameters for the decision making task.

To account for the abnormal errors, the minimax approach was found to be effective, and it was employed to design the decision function for correlation of measurements with tracked satellites. However, this approach is too rough to handle the parametric uncertainty. It is expedient to use more accurate techniques.

Let the vector of interfering parameters be $\boldsymbol{b}$. The updating of orbits, i.e., of the basic parameters $\boldsymbol{a} = (\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k)$, where k is the number of objects, is carried out on in such a way that in the course of data processing the interfering influence of parameters $\boldsymbol{b}$ will diminish.

An effective technique for solving this task is the adaptive Bayes approach, described by Repin and Tartakovsky.[6] This method suggests that the estimation $\hat{\boldsymbol{a}} = (\hat{\boldsymbol{a}}_1, \hat{\boldsymbol{a}}_2, \ldots, \hat{\boldsymbol{a}}_k)$ of the basic parameters is done by minimization in the complete set (basic and interfering) of parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ of the functional

$$\boldsymbol{\Phi}(\boldsymbol{X}, \boldsymbol{a}, \boldsymbol{b}) = \sum_{l=1}^{k} \boldsymbol{\Phi}_l(\boldsymbol{a}_l) \tag{19}$$

where $\boldsymbol{X}$ is the complete set of observations for all tracked objects and $\boldsymbol{\Phi}_l(\boldsymbol{a}_l)$ is the functional similar to Eq. (18), based on the observations of the lth object.

To find the solution we use the dynamic variant of the simple relaxation technique for minimization of nonlinear functions, where iterations in various parameters are replaced by iterations in time,[9] which means

$$\hat{\boldsymbol{a}}^{(s)} = \arg\min_{\boldsymbol{a}} \boldsymbol{\Phi}(\boldsymbol{X}_0, \boldsymbol{X}_1, \ldots, \boldsymbol{X}_s, \boldsymbol{a}, \hat{\boldsymbol{b}}_{s-1}) \tag{20}$$

$$\hat{\boldsymbol{b}}_s = \arg\min_{\boldsymbol{b}} \boldsymbol{\Phi}\big(\boldsymbol{X}_0, \boldsymbol{X}_1, \ldots, \boldsymbol{X}_s, \hat{\boldsymbol{a}}^{(s)}, \boldsymbol{b}\big) \tag{21}$$

where $\boldsymbol{X}_s$ is the set of measurements acquired during the interval τ_s where $\tau_0, \tau_1 \ldots, \tau_s$, are consequent nonintersecting time intervals.

The procedure is recurrent in time processing. Its most time-consuming part Eq. (20), performs independent processing of observations attributed to various objects according to the conventional mode using the estimation $\hat{\boldsymbol{b}}_{s-1}$ obtained earlier of the interfering parameters b. Joint processing of observations for various objects is done by a much simpler procedure, based on Eq. (21). The acquired estimates $\hat{\boldsymbol{b}}_s$ of $\boldsymbol{b}$ are used at the next step for the new portion of observations $\boldsymbol{X}_{s+1}$.

To create a realistic algorithm, we must specify the composition of vector $\boldsymbol{b}$. Vector $\boldsymbol{b}$ can be represented as $\boldsymbol{b} = (\boldsymbol{b}_1, \boldsymbol{b}_2)$, where $\boldsymbol{b}_1$ are parameters depicting prediction errors and $\boldsymbol{b}_2$ are present observation errors. Now consider each of the two components $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$. The major prediction errors are caused by atmospheric perturbations. They are defined by the errors in the prediction procedure model $\rho_m(h, t)$ of the atmosphere density $\rho(h, t)$, where h is altitude and t is time, and the behavior of the ballistic coefficient $k_b(t)$. It is assumed that $\rho_m(h, t)$ is given by the Russian model,[7] and $k_b(t) =$ const. The values $\rho(h, t)$ and $k_b(t)$ are incorporated into the equations of motion as multipliers. Thus, there is no need to evaluate them independently. It is sufficient to update the value $A = \rho k_b$ for each object individually. For convenience, instead of A we will use the parameter $s = A/\rho_m = k_b \cdot \rho/\rho_m$, which may be called the matching ballistic coefficient. It is one of the basic parameters to be assessed. The interfering component is the unknown root-mean-square deviation σ_s of its error or the value $\alpha = \sigma_s/s$ that is more suitable for the analysis.

The value α directly influences the observations' weights in the minimized function (18) because, for extensive propagating intervals, $\tilde{\sigma}_v \approx 0.5\alpha|\Delta T|N_{\text{pr}}^2$, where ΔT is the decline of orbital period per revolution, N_{pr} is the propagation interval (in revolutions), and $\tilde{\sigma}_v$ is the root-mean-square deviation of the prediction error (along the track). Overestimation of α will result in incomplete use of the observation data and in decrease of tracking accuracy that can consequently produce (if close objects exist) breaks of tracking. Underestimation of α (overly optimistic treating of propagation errors) can produce breaks of tracking even in the absence of close satellites.

Observation errors have normal and abnormal components. The unknown parameters of abnormal errors, for example, their maximum values, do not affect the quality of tracking, because their weights are set to zero in the process of abnormal components selection. Only the unknown parameters of the normal component distribution interfere: the mean (bias) m_x and the root-mean-square deviation σ_x. The errors in the parameters involved in orbit updating m_x and σ_x result in incorrect observations' weights in Eq. (18) that can produce breaks of tracking.

Thus, the vector $\boldsymbol{b}$ of interfering parameters is

$$\boldsymbol{b} = (\alpha, \boldsymbol{m}, \sigma) \tag{22}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{k_a})$ where k_a is the number of satellites affected by atmospheric drag and α_l is the root-mean-square deviation of relative error of the estimation of matching ballistic coefficient for the lth object ($l = 1, 2, \ldots, k_a$), and

$$\boldsymbol{m} = \big(\boldsymbol{m}_1, \boldsymbol{m}_2, \ldots, \boldsymbol{m}_{k_t}\big)$$

$$\sigma = \big(\sigma_1, \sigma_2, \ldots, \sigma_{k_t}\big)$$

where $\boldsymbol{m}_i$ and σ_i are the biases and the root-mean-square deviations of the observation errors for ith sensor, where k_t is the number of sensors.

Now consider the realistic algorithm based on Eqs. (20) and (21).

The estimation of parameter α for any atmosphere-affected satellite is fulfilled together with evaluation of the basic parameters $\boldsymbol{a}$. We solve the equation $f_1(\alpha) = 1$, where $f_1(\alpha)$ is the normalized sum of the components of functional (18) for the point of the minimum (functional for unit weight) for the most accurate parameters of the observations. The equation is solved iteratively. We start with evaluating parameters $\boldsymbol{a}$ by minimization of Eq. (18), where the weights of observations are calculated using the initial (obtained by the last updating) value α_{old}. Then we correct the estimate of α according to the formula $\alpha_{\text{new}} = \alpha_{\text{old}}\sqrt{}[f_1(\alpha_{\text{old}})]$ and the weights are recalculated. If the correction was essential, minimization of $\Phi(\boldsymbol{a})$ is repeated. Otherwise, the iterations are finished.

This procedure is used only for satellites substantially affected by atmospheric drag, i.e., for those having propagation errors (in most accurate observation components) essentially greater than the errors of the observations themselves.

Analysis of the characteristics of this procedure reveals that, in the course of tracking, the value α may vary within certain limits from $\alpha_{\min}$ to $\alpha_{\max}$. The weights of the observations vary respectively, and, hence, the effective memory of the algorithm changes. Thus, the procedure is adapting to the changes in the environment, that is, to the variations of atmospheric density and the satellite's ballistic coefficient. The magnitude of $\alpha_{\min}$ is satellite dependent and varies from several percent up to tens of percent. The magnitude $\alpha_{\max}$ may be up to hundreds of percent.

Evaluation of parameters $\boldsymbol{m}$ and σ for any sensor is done according to the standard formulas [immediately following from Eq. (21)] for evaluating the mean and the variance on the basis of a sample of differences between observed and estimated values of observations' parameters. Outdated observations are not used in these formulas. Thus, temporal variations of $\boldsymbol{m}$ and σ are accounted for by adapting to the real situation and reducing the interfering influence of these factors.

VII. Detection Process

Sections V and VI described the tracking algorithm that results in efficient catalog maintenance when the informativity condition is satisfied. For certain situations, this procedure is capable of solving the task even when the condition is not met. However, the handling of the observations produced by uncataloged satellites and the recovery of lost satellites are beyond the capabilities of this procedure. The rationale is the absence of sufficient a priori data to fulfill efficient correlation of the observations using the algorithm for preliminary correlation of observations to satellites.

The detection algorithm is free from these limitations. No requirements for the accuracy of the a priori data are posed here. These data may even be absent. The only requirement is that the informativity condition must be satisfied for some time. Then it will be possible to assign the uncorrelated observations to the satellites and determine their orbits with sufficient accuracy for further steady tracking.

In this sense we can say that the detection algorithm solves a more general task than the tracking procedure, and the tracking process is a particular case of detection process. However, the breaks of the informativity condition and the consequent employment of the detection procedure are of local character. Therefore, the detection procedure is only used occasionally whereas tracking is a continuous process.

The assignment χ_{tr} of measurements to objects satisfying the informativity condition is rather difficult in the detection process. An exhaustive search over all possible arrangements must be performed. This search is time consuming, and this is the main obstacle for implementation of the algorithm. Efficient organization of the exhaustive search is the essence of the procedure.

A. Complete Groups of Observations

For further considerations, the concept of a complete group of observations is important. This concept was mentioned in Sec. VI.D in the discussion of reliable decision making in the updating process. Let us consider this issue in more detail.

A set of measurements will be considered complete if the following two conditions are satisfied: 1) the reliability condition, where all of the measurements of the set inscribe into the orbit, determined using them and the probability that not all of the measurements belong to one satellite is small, and 2) the accuracy condition, where the orbit determined on the basis of the measurements of the considered set provides efficient correlation of future observations.

The complete groups of measurements are rather important. When at least one such group is discovered, the detection of the satellite that produced the measurements of this group is not a problem.

The minimal complete groups are of major interest, i.e., the groups comprising a sufficient minimum of observations. One measurement cannot form a complete group because its accuracy cannot ensure steady tracking. To make this situation possible, the maximal errors in velocity must be at least two orders of magnitude less then the errors of the existing Russian detection radars of decimeter band. At the same time, the reliability condition for one measurement is ideally satisfied automatically. Therefore, the availability of more precise measurements will essentially simplify the detection task, and the existence of the very precise measurement will make it trivial.

Two measurements, without rough errors and extended for a significant time interval (one revolution or more), produce a much more accurate orbit. However, they will not satisfy the reliability condition because the errors of propagation of one of them to the time of the second one are so large that a miscorrelation of alien observation may occur within this interval.

Finally, for three measurements without rough errors and with the time interval between any pair of them not less than one revolution, both conditions are satisfied. The accuracy condition usually is already satisfied for two boundary measurements. The reliability condition is also satisfied because the correlation gates determined by the orbit resulting from two boundary observations within the interval of their respective timings will have the magnitude of observation errors and will not let in the alien observations.

These considerations are certainly of plausible character and are not strict. However, they are verified not only by analytical studies, but by practice as well.

It should be mentioned that, though rare, situations exist when three measurements for different revolutions, but belonging to different satellites, inscribe into the created orbit. Thus, the modeling revealed that in the case when the boundary (regarding the timing) measurements belong to different fragments of a breakup they are often united into one orbit, and that occasionally a measurement from some third fragment can be inscribed into it. However, this situation is realistic only for the initial stage of the breakup, when shares of the observed fragments have not yet separated from one another, and all of them are orbiting within one tube. Later the probability of such an event becomes small.

Also rare, but real, are the situations when three or more measurements (for different revolutions) from one satellite inscribe into the orbit, but this orbit is not sufficiently accurate. The example is provided by the case when a small-sized object is observed by only one sensor with the regularity of one and the same (rather great) number of revolutions. These situations are also rather rare. However, in case such a situation occurs, further decision making may be disturbed.

The search for the minimal complete groups of observations, i.e., comprising three measurements, is the main task to be solved by exhaustive search over the uncorrelated measurements.

Assume that the minimal complete group is discovered and that the corresponding orbit is determined. Then according to the informativity condition (it must be supposed to be satisfied, otherwise it is too early to start the detection process and we are to resume accumulation of uncorrelated observations) only the own observations can inscribe into this orbit, being selected among all uncorrelated ones by the correlation procedure described in Sec. V. When all of the measurements corresponding to the determined object have been exhausted, the search for the complete group is performed again with the remaining uncorrelated observations, and the process continues until either no measurements remain, or those remaining do not constitute a complete group.

The resulting rather simple procedure of correlating the measurements to the objects is called the exhaustive technique. We will discuss some issues of its implementation subsequently.

B. General Structure of the Algorithm

The general structure of the detection algorithm is now discussed. The initial data for decision making are the set of uncorrelated observations $\boldsymbol{M}_{ut}$. The array of uncorrelated observations comprises the observations on the following: 1) arrived-in-space objects, generated by launches, separations, and breakups; 2) tracked satellites in case of breaks of tracking or a discrepancy between the actual observation errors and the accepted model; 3) unknown to the SSN satellites, resident in their orbits for a long time, but not observed before (note that such satellites may become observable due to commissioning of new sensors or the upgrading of the old ones); 4) tracked satellites, when there is a discrepancy between the actual observation or propagation errors and the model used for assignment of the observations to the satellites; and 5) nonexistent objects, which appear as a result of on-site determination of orbits based on noise registrations.

The observations that entered $\boldsymbol{M}_{ut}$ due to the first three causes constitute the set $\boldsymbol{M}_{ut}^{+}$. They contain useful unformation that is needed to solve the detection task. The last two groups of the observations constitute the set $\boldsymbol{M}_{ut}^{-}$. These measurements are the interfering background for solving the task.

The share of the observations in $\boldsymbol{M}_{ut}^{-}$ compared to the total set $\boldsymbol{M}_{ut}$ is rather important because the basic characteristics of the detection algorithm (the probability of miss, the frequency of false detections, and computation time) depend on this factor.

The probability of missing the own measurements in the course of preliminary measurements-to-satellites assignment is ≈ 0.001, and the share of uncorrelated measurements is ≈ 0.01 (Ref. 1). Hence, the share of the measurements in $\boldsymbol{M}_{ut}^{-}$ compared to $\boldsymbol{M}_{ut}$ is ≈ 0.1 (assuming that the share of the observations of nonexistent objects is small).

The detection task is being solved periodically in real time. Thus, the set $\boldsymbol{M}_{ut}$ can be divided into two parts: $\boldsymbol{M}_{ut}^{\text{old}}$ and $\boldsymbol{M}_{ut}^{\text{new}}$. The set $\boldsymbol{M}_{ut}^{\text{old}}$ comprises the previously acquired (old) observations that have been processed by the detection procedure earlier and were not incorporated into the previously determined orbits. The set M_{ut}^{new}

comprises the newly acquired observations that have not participated in the detection process yet.

The detection program could be initiated just after the arrival of only one observation, $\boldsymbol{x}_{\text{new}}$, in $\boldsymbol{M}_{ut}^{\text{new}}$. However, this is not done. The processing of the observation $\boldsymbol{x}_{\text{new}}$ is postponed for a revolution for the following reason. If we are processing a new launch or a breakup, in the course of approximately one revolution the measurements on all of the observable fragments of the launch or the breakup will enter $\boldsymbol{M}_{ut}$ and the detection problem will be solved more efficiently. In this sense a longer delay, for example, of one day would be even more effective. However, this will lead to a significant (quadratic in time) increase in the computation time causing a subsequent delay in data processing and other unwanted effects.

The detection program code starts running automatically, without involving any manual operations. When the a priori data on the certain event in space is available, the analyst can interfere with the detection process. Sometimes this interference is limited to just the input of these data, and further processing is performed automatically. However, sometimes the analyst interferes in the detection process directly, using the autonomous program codes operating in an interactive mode. We consider only the automatic mode.

Let the decision to run the detection program code be made for the new observation $\boldsymbol{x}_{\text{new}}$. Then the data processing scheme is as follows: 1) preliminary selecting of the triplets constituting the measurement $\boldsymbol{x}_{\text{new}}$; 2) determinating the primary orbit using the selected triplet; 3) selecting (from $\boldsymbol{M}_{ut}$) the measurements, inscribing into the orbit determined by the triplet, and updating of this orbit using the selected measurements; and 4) testing the reliability of the updated orbit.

If the search is successful and we manage to determine the orbit that satisfies the reliability condition, the search using the measurement $\boldsymbol{x}_{\text{new}}$ as a leading one is stopped. The leading measurement $\boldsymbol{x}_{\text{new}}$ is replaced with the the next new observation from the set $\boldsymbol{M}_{ut}^{\text{new}}$, and the search starts again. The measurements incorporated into the determined orbit do not participate in the next selection of the triplets. The process continues until all of the measurements constituting $\boldsymbol{M}_{ut}^{\text{new}}$ are processed.

Actually, not all of the new satellites (generated by a launch or a breakup) are observed equally, despite their orbits being close. The main causes are the different separation velocities, different sizes, and orientation. Thus, for certain objects the lack of the measurement data can occur, and the informativity condition will not be satisfied. Also even when the informativity condition is satisfied for all of the new objects, the solution provided by the already described draining procedure, generally speaking, does not coincide with the solution, which can be obtained using an exhaustive search. Hence, it is possible to generate an orbit on the basis of three measurements produced by different satellites.

To reduce the probability of generating such false orbits, the following technique is used. When we select from the set $\boldsymbol{M}_{ut}$ the measurements, inscribing into the next orbit determined using the triplet, all of the measurements from $\boldsymbol{M}_{ut}$ participate in the process (including those that are already fixed to the orbits determined previously on the basis of other leading measurements). The arising disputable measurements are assigned to the orbits closest to them.

When the processing of all of the new measurements is completed, some measurements may be taken away from certain orbits by other ones. Therefore, the robbed orbits are again tested for reliability.

The measurements assigned to reliable orbits are removed from $\boldsymbol{M}_{ut}$, and these orbits enter the process of preliminary tracking. During this phase, the tasks of orbital and satellite identification must be solved (as mentioned in the Introduction). The next four sections describe the aforementioned parts of the automatic algorithm.

C. Preliminary Selection of Triplets

Each day ≈400 observations are added to the uncorrelated measurements' file.[10] The time intervals needed to detect certain satellites may extend to 1–2 months. Thus, the number of observations accumulated in the file of uncorrelated measurements is on the order of thousands. Hence, the search for the triplets of measurements that constitute complete groups is not a simple task. The task is not easy because the possible number of the combinations of three measurements that include the considered observation $\boldsymbol{x}_{\text{new}}$ is of the order of 10^6–10^8. Thus, it is expedient to exclude immediately the measurements that surely have no relation to the satellite that produced the observation $\boldsymbol{x}_{\text{new}}$.

The parameters i, Ω, and T, which can be rather accurately calculated using the measurement [the errors are rather small (see Sec. II) and the evolution with account of these errors is also rather simple] are used for this selection.

After this selection, the remaining observations satisfy the conditions: $|i_n - i| < c_i$, $|T_n - T| < c_T$, and $|\Omega + (t_n - t)\dot{\Omega} - \Omega_n|_{\text{mod}2\pi} < c_\Omega$, where the parameters with the subscript n correspond to $\boldsymbol{x}_{\text{new}}$ and without the subscript correspond to any measurement from $\boldsymbol{M}_{ut}$.

Here c_i, c_Ω, and c_T are the selection gates. Their values depend on the sensors that produced the involved measurements, on the radar cross-sectional measurements (included with the observations of certain sensors), and on the difference in time betwen the observations and the parameters of the satellite. The ranges for the gates c_i, c_Ω, and c_T are respectively 1–5 deg, 1–100 deg, and 5–250 min. Parameters of these relationships are chosen experimentally. The requirement is that the probability of excluding the measurement produced by the same satellite that produced $\boldsymbol{x}_{\text{new}}$ must be less than 0.1.

The set of the remaining measurements can be called the group of the measurement $\boldsymbol{x}_{\text{new}}$. Only the observations of this group participate in the search for triplets. The number of measurements in the group of $\boldsymbol{x}_{\text{new}}$ is much lower than the whole array of uncorrelated measurements. However, even for a typical situation when no new objects are observed in space, this number may exceed 100. Therefore, it is reasonable to perform the preliminary selection of triplets, without using precise prediction and the minimization of functional (18), within the group of $\boldsymbol{x}_{\text{new}}$.

To select the triplets, surely not corresponding to one and the same satellite, we use the time parameter or, more accurately, the time t_u when the satellite reaches a certain argument of latitude u within the revolution corresponding to the observation. Parameter t_u can be calculated from measurement with insignificant methodical error using simple Keplerian formulas. The value of u, used in the calculations for all tested measurements, is the mean of the arguments of latitiude for these observations.

Selection is performed comparing the value t_u corresponding to the middle (with respect to timing) observation of the triplet to its estimate $\hat{t}_u$ calculated using two boundary observations (the left, with mininum timing, and the right, with maximum timing). The formula for calculating $\hat{t}_u$ is

$$\hat{t}_u = p t_u^{(+)} + (1-p) t_u^{(-)}, \qquad p = M/N \tag{23}$$

where $t_u^{(+)}$ and $t_u^{(-)}$ are the times when the satellite reaches u for the revolutions, corresponding to the boundary (right and left) measurements of the triplet; N is the number of the revolutions between the boundary measurements; and M is number of the revolutions between the left and the middle observations ($N > 1$, $M < N$).

The condition for selection is as follows:

$$|t_u - \hat{t}_u| < c_u \tag{24}$$

where $c_u = \delta t_u + p\delta t_u^{(+)} + (1-p)\delta t_u^{(-)}$ is the gate depending on the errors δt_u, $\delta t_u^{(+)}$, and $\delta t_u^{(-)}$ in the determination of t_u, $t_u^{(+)}$, and $t_u^{(-)}$.

Actually, the number of the revolutions N between the two boundary observations is determined within the interval $(\bar{N} - k_{ud}, \bar{N} + k_{ud})$, where $\bar{N}$ is the estimate of N on the basis of boundary measurements and k_{ud} is the uncertainty factor. Thus, inequality (24) is to be tested for each $N \in (\bar{N} - k_{ud}, \bar{N} + k_{ud})$.

The values $\bar{N}$ and k_{ud} are calculated using the formulas

$$\bar{N} = \left[\frac{\left(t_u^{(+)} - t_u^{(-)}\right)}{\left(\tilde{p}T^{(-)} + (1-\tilde{p})T^{(+)}\right)}\right]$$

$$k_{ud} = \left[\frac{\bar{N}\left|\tilde{p}\delta T^{(-)} + (1-\tilde{p})\delta T^{(+)}\right|}{\left(\tilde{p}T^{(-)} + (1-\tilde{p})T^{(+)}\right)}\right] \tag{25}$$

where $[A]$ is the integer of A (rounded off); $T^{(-)}$, $T^{(+)}$, $T^{(-)}$, and $T^{(+)}$ are the orbital periods calculated using boundary observations and their errors; and $\tilde{p}$ is the weighing coefficient, which for known variances $\sigma^2_{T(+)}$ and $\sigma^2_{T(-)}$ of the errors $\delta T^{(+)}$ and $\delta T^{(-)}$ in the determination of the values of $T^{(+)}$ and $T^{(-)}$ can be calculated using the formula $\tilde{p} = \sigma^2_{T(+)}/(\sigma^2_{T(+)} + \sigma^2_{T(-)})$.

If $k_{ud} = 0$, there is no uncertainty and $N = \bar{N}$. As we have already mentioned in Sec. II, for meter-sized satellites $\delta T \approx 1$ min. Thus, the uncertainty in the number of revolutions occurs when $N \approx 50$ (3–4 days). In practice this situation is rare. However, for small-sized objects, the uncertainty arises even for $N \approx 10$, and this situation is typical.

The triplets are processed in the following way. One of the observations is known. This is the observation $\boldsymbol{x}_{\text{new}}$ (at the right end of the interval). The left observations are selected according to increasing time, starting from the most remote (the oldest). For the fixed left measurement, the intermediate measurements are selected under the condition that all three measurements must belong to different revolutions. Finally, when the left and the intermediate measurements are fixed, the search through all possible values of $N \in \{\bar{N} + i, i = 0, \pm 1, \pm 2, \ldots, \pm k_{ud}\}$ is performed. The selected triplet is forwarded to the primary orbit determination procedure along with the obtained estimation of the number of revolutions between the boundary measurements. If the three selected measurements remain connected for k different values of N, the calculation of the primary orbit is performed k times.

The values c_u and k_{ud} are the basic parameters of the algorithm. They must be calculated under the requirement that the probability of missing the detection must be sufficiently small. (The specific value of the tolerable level for this probability depends on the importance of the object. However, its value is always less than 0.1.) However, the values of c_u and k_{ud} calculated under this assumption, for rough and significantly distant in time measurements, may lead to false detections. When the group $\boldsymbol{x}_{\text{new}}$ contains many measurements, the computation time may be unacceptably long. Therefore, in practice the limits $c_{u,\max}$ and $k_{ud,\max}$ are posed for these parameters.

The determination of $c_{u,\max}$ and $k_{ud,\max}$ was not a simple task because the increase of these parameters produces an abrupt increase in computation time. A thorough analysis performed using the real data and modeling resulted in an acceptable compromise. The chosen values of $c_{u,\max}$ and $k_{ud,\max}$ are the functions of the number of the measurements in the group of the measurements $\boldsymbol{x}_{\text{new}}$, the number of the new (not used previously in the detection process) measurements in this group, and the density of the uncorrelated measurements in the phase–time domain $D = \{i, \Omega, T, t\}$.

When most of the satellites generated by the last events are already detected (the usual situation), $c_{u,\max} \approx 1$ min and $k_{ud,\max} = 40$.

D. Primary Determination of Orbit

If the triplet of observations have passed the preliminary selection the attempt to determine the primary orbit is made. To solve this task, the functional (18) is minimized, where $m = 3$, $r = 1$, and $\bar{\boldsymbol{K}}_p = 0$. Let us discuss this procedure.

For the calculation of $\boldsymbol{\Phi}$ at the point $\boldsymbol{a}$, the analytical prediction procedure is used that accounts for all of the zonal harmonics of the geopotential through the sixth, the second tesseral, and the static model of the atmosphere with the parameters, depending on the index of solar activity $F_{10.7}$ (Ref. 3, algorithm A).

For the minimization of $\boldsymbol{\Phi}(\boldsymbol{a})$, the combination of Gauss–Newton and the steepest descent techniques (described in Sec. VI.C) is used. The matrices of the partial derivatives of the observation parameters with respect to the orbital parameters are calculated using the analytical formulas that include the second zonal harmonic and the static model of the atmosphere. The matrices are recalculated each time, when the point for which the value of the functional is calculated changes.

In $\boldsymbol{\Phi}(\boldsymbol{a})$, the a priori data $\boldsymbol{a}_a$ and $\boldsymbol{P}_a$ are incorporated. They assist the convergence of the process and also can be used as the means for the control. The first six componenets of the vector $\boldsymbol{a}_a$ are the parameters of the initial approximation, and $s = s_a$, where $s_a = 10^{-10}$ km^3/s^{2}kg. The weights p of the squares of the residuals with the a priori data have the values corresponding to the errors of the initial approximation. The parameter s is not updated (the specific choice of p_s is used).

We consider that the primary orbit is determined when the iterations converge to the point $\hat{\boldsymbol{a}}$, $\boldsymbol{\Phi}(\hat{\boldsymbol{a}}) < c_\Phi$ (in addition to this check, the value of the functional, calculated using only the three or four most accurate parameters of each of the measurements, is compared with the threshold $\tilde{c}_\Phi$) and the obtained solution dominates the competing ones. Actually, the absence of a dominating solution is a rare case even when the time interval between the neighboring measurements is about 1 month. This situation occurs for a satellite observed by only one sensor under the specific condition that the passes through the field of view repeat regularly with an interval of one or more revolutions. Normally the observations arrive irregularly, and the uncertainty in the number of the revolutions can be removed. The competing orbits are the orbits determined on the basis of the same triplet for the other values of N (that have passed the preliminary selection). Dominating means that the orbit fits the measurements significantly better, i.e., the value of $\boldsymbol{\Phi}(\hat{\boldsymbol{a}})$, compared to the competitors, is not less than k_Φ times smaller. The parameters $\tilde{c}_\Phi$, c_Φ, and k_Φ are chosen experimentally ($\tilde{c}_\Phi$ ranging from 10 to 200, c_Φ from 100 to 10,000, and k_Φ from 10 to 100). They depend on the perigee altitude, orbital eccentricity, and the maximum density of the observations of the group $\boldsymbol{x}_{\text{new}}$ in the phase–time domain. The key aspect of the algorithm is the technique of generating the initial approximation $\boldsymbol{a}_0$ for the vector $\boldsymbol{a}$ in the process of the minimization of $\boldsymbol{\Phi}(\boldsymbol{a})$ because its accuracy determines the convergence rate for the iterative process.

The parameters $\tilde{\boldsymbol{a}}$ determined using one measurement can be used as the initial approximation. If this observation has no rough errors, and other measurements participating in the minimization are located in the neighboring revolutions, the iterative process of minimization, as a rule, converges to the absolute minimum. However, usually the real situations are different; convergence does not occur or the process converges to an extraneous minimum $\boldsymbol{\Phi}(\boldsymbol{a})$.

The major factor hampering the convergence is the significant error in the orbital period, calculated using the measurement. The following technique can be used to reduce the errors in the orbital period for the initial approximation.

The process of triplets' selection produces the draconic period T_Ω

$$T_\Omega = \frac{\left(t_u^+ - t_u^-\right)}{N} \tag{26}$$

Note that the accuracy of calculating the draconic period using Eq. (26) is very high. Even for $\bar{N} = 1$, the error is several times lower than for the calculations using one measurement. With the increase of $\bar{N}$, the error linearly decreases to the value of $\approx$0.01 min.

Further, T_Ω can be transferred to the osculating T using the known formulas, including the second zonal harmonic of the geopotential. By using T, the value of L is calculated, and then in the initial approximation of the vector $\tilde{\boldsymbol{a}}$ the value $\tilde{L}$ is replaced with L. However, studies, based on real data on small-sized satellites, revealed that this operation does not ensure reliable convergence, i.e., not less than 90%. We need to find the way to reduce the errors of other parameters as well.

The investigation demonstrated that, in this situation, any parameter of vector $\tilde{\boldsymbol{a}}$ affects the convergence. Thus, the problem of the choice of the initial approximation virtually becomes the equivalent of the primary task of orbit determination. The obstacle can be removed by the transition to the system of the parameters of the observation $\boldsymbol{x}_{\text{new}}$.

In this system the vector of orbital parameters has the form

$$\boldsymbol{x} = (D, \varepsilon, \gamma, \dot{D}, \dot{\varepsilon}, \dot{\gamma}, s) \tag{27}$$

The studies demonstrated that when the minimization of $\boldsymbol{\Phi}[\boldsymbol{a}(\boldsymbol{x})] = \boldsymbol{\Psi}(\boldsymbol{x})$ is performed not with respect to the orbital elements, but with respect to the radar parameters of the vector $\boldsymbol{x}_{\text{new}}$, using the $\boldsymbol{x}_{\text{new}}$ itself as the initial approximation, then for the cases in which the convergence problems really arise, i.e., in the detection of the small-sized satellites, the convergence is mostly influenced by the errors in the angular velocities $\dot{\varepsilon}$ and $\dot{\gamma}$. The parameters D, ε, γ, and $\dot{D}$ are already determined with sufficient accuracy. Thus, we have already obtained four of the six parameters needed for the initial approximation. However, we have the fifth parameter T, which is known with accuracy sufficient to ensure the convergence and independence from the parameters D, ε, γ, and $\dot{D}$. Thus, the number of the satellite's parameters known with sufficient accuracy is five, and there is only one rough parameter.

The parameters $\dot{\varepsilon}$, $\dot{\gamma}$, and T are in a functional relationship with one another. This function is rather simple because the semimajor axis a and the square of the velocity v^2 are calculated from T and v^2 is a quadratic function of $\dot{\varepsilon}$ and $\dot{\gamma}$.

Thus, we have only one rough parameter, which can be determined by the minimization of $\boldsymbol{\Psi}(\boldsymbol{x})$ in this parameter under the fixed values of other parameters. This minimization is not difficult. The classical combination of Gauss and the steepest descent techniques (quickly converging) is used. For the initial approximation the radar parameters of the vector $\boldsymbol{x}_{\text{new}}$, one of the values $\dot{\varepsilon}$ or $\dot{\gamma}$ is replaced with its value calculated from T. As a result we obtain the missing sixth parameter of the initial approximation.

E. Calculation of Updated Orbits

Apart from the three selected observations, the file of the uncorrelated measurements can comprise the other measurements that inscribe into the primarily determined orbit. Therefore, they are bailed out from the file of uncorrelated observations. The procedure described in Sec. V is used for this operation.

Then the orbital parameters are calculated using the algorithm described in Sec. VI. All of the available measurements are used (the three initial ones and the others correlating with the orbit). This procedure is distinct from the algorithm used for the primary determination of the orbit on the basis of three observations. The number of the used observations can be more than three, the weights of the measurements are calculated taking into account the propagation errors along with the observation errors, the initial approximation is formed using the primary orbit, the more precise analytical algorithm AP [or numerical–analytical algorithm (NA)[11]] is used for propagation, the parameter s is updated, and the selection of the abnormal and the alien measurements is fulfilled. Note that it is very important to have the value of the weight P_{s_a} of the a priori value s_a of the matching ballistic coefficient, corresponding to its real error, which for the satellites with significant sailing properties can exceed s_a by 2–3 orders of magnitude. After one updating, the resulting orbit is normally more reliable and accurate. Therefore, the described procedures are repeated for this orbit. The process continues until the set of the measurements correlated to the orbit is empty.

The obtained updated orbits are tested for reliability. If the calculations of the updated orbit did not produce an orbit reliable according to the criteria given in Sec. VI.D, it is considered that the primary orbit is not generated, and the search involving the measurement $\boldsymbol{x}_{\text{new}}$ proceeds further.

When the processing of all of the measurements is completed, the measurements of the determined and updated orbits may be rearranged (see Sec. VII.B). Thus, the resulting orbits again are tested for reliability, and only those orbits that have passed this test succesfully are considered detected. The reliability criteria is described in Sec. VI.D. Here we will clarify how the completeness condition is tested.

We test the residuals between the observations and the determined orbit; the number of the measurements; which residuals allow them to be inscribed into the orbit, i.e., the absolute value of the residual Δ is less than the threshold c_Δ; and the number of the revolutions where the inscribed measurements exist.

The parameters of the decision function (the thresholds c_Δ for residuals in the different parameters of the observation, the threshold c_{obs} for the number of inscribed observations, and the threshold c_{rev} for the number of revolutions with inscribed measurements) are chosen to satisfy the requirement for the frequency of orbit updating. It must be not less than 0.8. The orbit is considered updated if either its parameters are refined using new measurements or it is identified with the newly detected orbit.

Parameters c_Δ, c_{obs}, and c_{rev} depend on the sensor, the interval of radar tracking, radar cross section, perigee altitude h_p, orbital eccentricity e and the decline of orbital period per revolution ΔT of the satellite, and the density of the observations M_{ut} in the phase–time domain.

F. Orbital Identification

Before making the decision that a new object is detected, we must be sure that its orbit is not the orbit of a previously cataloged, but now lost satellite. This is the task of the identification algorithm.

Orbital identification, as well as the assignment of the measurements to the cataloged satellites, is the task of testing multialternative statistical hypotheses. Thus, the efficient solution of the identification task is possible when the informativity condition is satisfied for the set of possible hypotheses. This means that, with regard to the parameters for decision making, the distance to the true hypothesis must be essentially smaller than the distances to the competing ones. The errors in orbital parameters are several orders of magnitude less than the maximal observation errors. Thus, the identification task is more likely to be solved efficiently than the task of observations' correlation. In particular, it is possible to solve the task efficiently for significantly longer time intervals. The same factor determines the rather simple structure of the algorithm.

The decision is made by comparing the differences $\delta a = \boldsymbol{a}_1 - \boldsymbol{a}_2 = (\delta a_1, \delta a_2, \ldots, \delta a_7)$ in the orbital parameters with the thresholds $\Delta\boldsymbol{a} = (\Delta a_1, \Delta a_2, \ldots, \Delta a_7)$, depending on the parameters of the orbits and the time intervals between them. (The expressions for these thresholds are derived in Ref. 12.) For calculating the differences, the orbit of the cataloged satellite is propagated to the epoch of the detected one. The uncertainty of identification is removed using the value φ of the normalized distance between the orbital parameters used in identification, where

$$\varphi^2 = \sum_{i=1}^{7} \frac{(\delta a_i)^2}{(\Delta a_i)^2}$$

The parameters of the decision function are chosen empirically on the basis of extensive data stored for several years. The primary requirement was to obtain a small probability of false identification. For distinction, note that the parameters of measurements-to-objects correlation procedure are chosen to provide small missing probability.

To reduce the amount of calculations, the rough selection of cataloged objects surely not identifying with the new orbit is done. Parameters i, Ω, and T are used assuming that i is constant and that the evolution of Ω and T is linear. The formulas for the gates are derived in Ref. 12.

Identification of any detected orbit is performed with all of the cataloged satellites, both tracked and not tracked (having the flag prohibiting correlation). When the identification decision is made, the orbital parameters and observation data for the satellite are renewed, and the correlation prohibiting flag is removed. Unidentified orbits are cataloged as new satellites. They will further participate in catalog maintenance together with other cataloged objects. However, their final destiny is determined later in the course of preliminary tracking, supervised by the analyst.

G. Working in Complicated Situations

The most complicated situation for the automatic detection program is a satellite breakup. Breakup events are rather rare. In recent years the annual number of breakups generating many observable objects has been less than one. On 3 July 1996 the rocket-body "Pegasus" (international designator 94-29-2) exploded, and several hundred fragments were observed.

When a breakup occurs, the number of uncorrelated measurements increases drastically. The majority of the observable fragments are small sized, and the orbits of the fragments are close. Thus, the majority of the measurements produced by the breakup after the preliminary selection fall into one group. The breakup produces a peak load for the detection algorithm. The efficiency of the algorithm is rather low, especially when the breakup occurs at a low altitude because during the initial stage of the breakup the informativity condition is disturbed more often than it is satisfied. This is the cause for delay in detection. Some of the observed objects even reenter prior to their detection. However, the proper use of the breakup data improves the performance of the detection algorithm. Let us consider the relevant techniques.

Detection of the breakup event is possible on the basis of assessment (monitoring) of the time–spatial density of the uncorrelated observations (using parameters i, Ω, and T). In the case of a breakup, this density abruptly increases. When the breakup event is detected, the parameters of the decision function for testing the primarily determined orbits for reliability automatically change within the area of the breakup. The reliability criteria become more strict, thus ensuring an acceptable level for false orbit detection.

Normally the satellite that suffered the breakup becomes lost. In this event, its catalog number is present in the measurements of the group that entered the detection process. Thus, this satellite can be discovered. When there is no break of tracking and the new fragment is tracked instead of the parent satellite, the preliminary discovery of the parent can be performed using the analysis of the residuals betwen the uncorrelated measurement and the orbits of relevant satellites and the number of the alien measurements selected in the process of updating their orbits.

When at least one new object is detected, the chances of determining the parent more reliably (in case it was previously tracked) and determining the time of the breakup increase. These operations are performed comparing the calculated positions of all of the detected satelites (including the parent) for various times. Some of the computation issues are discussed in Ref. 11.

If the parent satellite and the time of the breakup are known the number of observed objects and their orbital parameters can be assessed. To do this, the orbital parameters, calculated using any of the uncorrelated measurements assigned to the breakup, are updated using the data at the time of the breakup. Then the twins corresponding to other measurements produced by already determined fragments of the breakup are removed.

The orbits calculated in this way are rather accurate. However, the steady tracking of these orbits is often not possible due to the high spatial density of observed objects. This is why they are not transitioned to the regular tracking process. In the course of further separation of the fragments, they are detected in the routine manner and are identified with the orbits obtained previously. Then the tracking of these satellites begins.

VIII. Conclusions

The study presented leads to the following major conclusion. The methodologically unified approach to the whole problem is very useful for the development of the complex of algorithms for the monitoring of satellites' orbital motion.

This approach assumes the posing of the whole task in the most general way: obtaining the principal solution and the informational analysis of this solution taking into account the characteristics of the observed objects and the performed measurements. This is the basic principle of the present work. To our understanding, the methods of statistical decision theory under the conditions of a priori uncertainty are the best technique for this task. Thus, they are used as the basis for the entire complex algorithm as well for its components.

The complex algorithm is rather sophisticated. In particular, 1) in catalog maintenance procedures, the principal calculations with the observations are performed in the radar coordinate system; 2) all of the decision-making processes account for the possibility of abnormal errors in the observations; 3) the primary determination of orbits uses the time-consuming exhaustive search for the triplets of uncorrelated measurements; and 4) the joint processing of the observations is used at all of the stages in the final decisions regarding the assignment of the measurements and determination of the orbital parameters.

Rejection of some of these principles for the sake of the simplicity of the algorithm leads a decrease in the efficiency of catalog maintenence.

Although the described approach is essentially universal and can be used for other tasks, in particular for catalog maintenance in other informational systems, the described algorithms are directly related to the features of the Russian network of sensors. For other systems, the efficient solutions can be attained using different procedures. Thus, for the systems with larger informational capabilities, for example, for the U.S. SSN, similar tasks can probably be solved using simpler techniques.

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